Math 567: Introduction to Coding Theory

Homework 4: Due Wednesday, March 20

Problem 1.1. For $i = 1, 2$, let $\mathcal{C}_i$ be a linear $[n_i, k_i, d_i]$-code with generator matrix $\mathcal{C}_i$ and parity check matrix H_i. Define

$$\mathcal{C}_1 \oplus \mathcal{C}_2 = \{(c_1, c_2) : c_1 \in \mathcal{C}_1, c_2 \in \mathcal{C}_2\}.$$

1. Show that $\mathcal{C}_1 \oplus \mathcal{C}_2$ is an $[n_1 + n_2, k_1 + k_2, \min(d_1, d_2)]$ code.
2. Show that the generator matrix of $\mathcal{C}_1 \oplus \mathcal{C}_2$ is $\begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}$.
3. Show that the parity check matrix of $\mathcal{C}_1 \oplus \mathcal{C}_2$ is $\begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}$.

Problem 1.2. For $i = 1, 2$, let $\mathcal{C}_i$ be a linear $[n, k_i, d_i]$-code and define

$$\mathcal{C} = \{(u, u + v) : u \in \mathcal{C}_1, v \in \mathcal{C}_2\}.$$

1. Show that $\mathcal{C}$ is an $[2n, k_1 + k_2, \min(2d_1, d_2)]$ code.
2. Show that the generator matrix of $\mathcal{C}_1 \oplus \mathcal{C}_2$ is $\begin{bmatrix} G_1 & G_1 \\ 0 & G_2 \end{bmatrix}$.
3. Show that the parity check matrix of $\mathcal{C}_1 \oplus \mathcal{C}_2$ is $\begin{bmatrix} H_1 & 0 \\ -H_2 & H_2 \end{bmatrix}$.

Problem 1.3. Let $\mathcal{C}$ be an $[n, k, d]$ code over $\mathbb{F}_2$ and $\mathcal{C}^*$ the code resulting from puncturing $\mathcal{C}$ at the ith coordinate.

1. If G is the generator matrix for $\mathcal{C}$ then the generator matrix for $\mathcal{C}^*$ is G minus the ith column, where if there are two identical rows in the resulting matrix we delete one of them, and if there is a zero row we delete that as well.
2. If $d > 1$ then is an $[n - 1, k, d^*]$ code, where $d^* = d - 1$ if $\mathcal{C}$ has a minimum weight codeword with a nonzero ith coordinate and $d^* = d$ otherwise.

Problem 1.4. Let $\mathcal{C}$ be a cyclic code with generating polynomial $g(x) = g_{n-k}x^{n-k} + \cdots + g_1 x + g_0$. Then $\mathcal{C}$ has $\{g(x), xg(x), \ldots, x^{k-1}g(x)\}$ as a basis. In particular the dimension of $\mathcal{C}$ is k. The generator matrix of $\mathcal{C}$ is

$$G = \begin{pmatrix} g_0 & g_1 & \cdots & g_{n-k} & 0 & \cdots & 0 \\ 0 & g_0 & \cdots & \cdots & g_{n-k} & & \\ \vdots & \vdots & \ddots & & & \ddots & \\ 0 & 0 & \cdots & g_0 & g_1 & \cdots & g_{n-k} \end{pmatrix}$$

Problem 1.5. Construct a Reed-Solomon code over $\mathbb{F}_3$ of length 3 and dimension 2.

1. List all of the codewords in your code.
2. What is the minimum distance of your code?
3. Give an explicit proof (without referring to anything we did in class) that your code is linear.