# Math 567: Introduction to Coding Theory

Homework 6: Due Friday, April 5

**Problem 1.1.** Compute the weight enumerator of each of the following codes:

1. The Hamming $[7,4]$ code.
2. The $[7,4]$ binary cyclic code with generating polynomial $g(x) = x^3 + x + 1$.

**Problem 1.2.** Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be two binary linear codes. Show that

$$W_{\mathcal{C}_1 \oplus \mathcal{C}_2}(x, y) = W_{\mathcal{C}_1}(x, y) W_{\mathcal{C}_2}(x, y).$$

**Problem 1.3.** Let $\mathcal{C}$ be the $[5,2]$ binary code generated by

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

1. Find the weight distribution of $\mathcal{C}$.
2. Find the weight distribution of $\mathcal{C}^{\perp}$.
3. Verify your result in 2. by listing the vectors in $\mathcal{C}^{\perp}$.

**Problem 1.4.** Let $G$ be a finite cyclic group. Show that the *double dual* $\widehat{\widehat{G}}$ (the dual group of the dual group of $G$) is isomorphic to $G$; that is, $G \cong \widehat{\widehat{G}}$.

**Problem 1.5.** Let $G$ be a finite abelian group and $f, g : G \to \mathbb{C}$ be functions. Show that

$$\sum_{\psi \in \widehat{G}} \widehat{f}(\psi) \overline{\widehat{g}(\psi)} = \frac{1}{|G|} \sum_{x \in G} f(x) \overline{g(x)}.$$