Math 567: Introduction to Coding Theory

Homework 7: Due Monday, April 22

Problem 1.1. Let $\mathcal{C}$ be a binary $[n, k, d]$ code and $\Lambda(\mathcal{C})$ be the lattice associated to $\mathcal{C}$ by Sloane's Construction A. Show that $\det(\Lambda(\mathcal{C})) = 2^{n-2k}$.

Problem 1.2. Let Λ be a rank n lattice in $\mathbb{R}^n$ with fundamental parallelepiped $P(\Lambda)$. Show that every point $u \in \mathbb{R}^n$ can be written uniquely as $u = x + y$ for some $x \in \Lambda$ and $y \in P(\Lambda)$. (Here we are adopting the convention that if Λ has basis $\lambda_1, .., \lambda_n$ then $P(\Lambda) = \{\sum_{i=1}^n x_i\lambda_i : 0 \leq x_i < 1\ \forall i\}$.)

Problem 1.3. Let Λ be a rank n lattice in $\mathbb{R}^n$ with fundamental parallelepiped $P(\Lambda)$ and let $B_r = \{x \in R^n : \|x\| \leq r\}$ be the ball of radius $r > 0$. Show that

$$\lim_{r\to\infty} \frac{|\Lambda \cap B_r|}{\mathrm{Vol}(B_r)} = \frac{1}{\det(\Lambda)}.$$

Problem 1.4. Prove that if Λ is an even unimodular lattice of rank n then $n \equiv 0 \pmod 8$.

Problem 1.5. Describe the set of flat tori $\mathbb{R}^n/\Lambda$ which are isospectral to the 1-torus $\mathbb{R}/\mathbb{Z}$.